2-Sample Confidence Intervals for the Difference in Means

Requirements for complete responses to free response questions that require a 2-Sample Confidence interval for means:

1. Identify the population parameters of interest, the type of inferential procedure to be used, and the confidence level.
   - A confidence interval is used to estimate the value of an unknown *parameter*.
   - The *confidence level* gives the percent of the intervals produced by the procedure you used that capture the true value of the parameter.
   - The population parameters of interest must be defined in context of the problem.
2. State and verify whether the conditions (assumptions) needed for this procedure are met.
   - The conditions for a 2-sample confidence interval for the difference in means are as follows:
     - Both samples must be independent random samples from the populations of interest or, in an experimental setting, random assignment to treatments must be established. You must state this condition and verify whether or not this condition is met.
     - The samples must be large. To verify this condition, check that $n_1 \geq 30$ and $n_2 \geq 30$. You must show your work to receive credit for verifying this condition. The samples must be large. To verify this condition, check that $n_1 \geq 30$ and $n_2 \geq 30$. You must show your work to receive credit for verifying this condition. If the sample sizes are small, you must show a graph of both sample data sets and state how each graph shows whether it is reasonable to assume each population is normally distributed or not.
     - The observations within each sample are independent. When sampling without replacement, each of the two populations must be at least 10 times greater than the sample size.
3. Write the formula for the confidence interval and determine the interval.
   - Every confidence interval computation follows the same general form (given on the formula sheet):

     $$(\text{Sample statistic}) \pm (\text{distribution critical value}) \times (\text{standard deviation of statistic})$$
   - Determine the degrees of freedom using the smaller of $n_1 - 1$ or $n_2 - 1$.
4. Interpret the interval in the context of the problem.
   - Include a statement that links the *confidence level* to the interval.
   - When you interpret a confidence interval, be very careful that you do not associate the probability with the parameter. Keep in mind that the value of the parameter does not vary, so there is no chance associated with its value. The sample is what varies!
   - It is risky to try to interpret a confidence interval in a creative way. Learn a good confidence interval interpretation and stick to it! A suggested writing template for the interval is:

     We are _____% confident that the true difference in mean "context of problem" for "first group context" and "second group context" is between _______and _________(units).

If you are asked to interpret the confidence level, the following is an appropriate interpretation of confidence level:

> _____% of the intervals produced using this method will capture the true difference in mean "context of problem" for "context of first group" and "context of second group."

Confidence Intervals for Matched Pairs:

When the observations are collected in pairs or the observations in one group are related to the observations in the other, a confidence can be constructed by determining an interval for the mean difference. The conditions for this type of interval are the same as a one-sample *t*-interval, using the differences that must be independent of each other.

The formula is: $\bar{d} \pm t^{*}_{n-1} \frac{s_d}{\sqrt{n}}$, where $\bar{d}$ is the mean difference and s_d is the standard deviation of the differences.

Multiple Choice Questions:

1.

Ten students were randomly selected from a high school to take part in a program designed to raise their reading comprehension. Each student took a test before and after completing the program. The mean of the differences between the score after the program and the score before the program is 16. It was decided that all students in the school would take part in this program during the next school year. Let μ_A denote the mean score after the program and μ_B denote the mean score before the program for all students in the school. The 95 percent confidence interval estimate of the true mean difference for all students is $(9, 23)$. Which of the following statements is a correct interpretation of this confidence interval?

A. $\mu_A > \mu_B$ with probability 0.95.
B. $\mu_A < \mu_B$ with probability 0.95.
C. μ_A is around 23 and μ_B is around 9.
D. For any μ_A and μ_B with $(\mu_A - \mu_B) \geq 14$, the sample result is quite likely.
E. For any μ_A and μ_B with $9 < (\mu_A - \mu_B) < 23$, the sample result is quite likely.

2.

A student working on a history project decided to find a 95 percent confidence interval for the difference in mean age at the time of election to office for former American Presidents versus former British Prime Ministers. The student found the ages at the time of election to office for the members of both group, which included all of the American Presidents and all of the British Prime Ministers, and used a calculator to find the 95 percent confidence interval based on the *t*-distribution. This procedure is <u>not</u> appropriate in this context because

A. the sample sizes for the two groups are not equal
B. the entire population was measured in both cases, so the actual difference in means can be computed and confidence interval should not be used
C. elections to office take place at different intervals in the two countries, so the distribution of ages cannot be the same
D. ages at the time of election to office are likely to be skewed rather than bell-shaped, so the assumptions for using this confidence interval formula are not valid
E. ages at the time of election to office are likely to have a few large outliers, so the assumptions for using this confidence interval formula are not valid

3.

Researchers believe that both the Galapagos turtles and sea turtles have evolved into giant tortoises even though one species lives on the tropical island and the other lives in the sea. The summary statistics for weight for each type of tortoise is listed below.

	$\bar{x}$	s	n
Galapagos	550	54.2	23
Sea Turtles	460	68.3	15

Which of the following is the appropriate method to use to construct a 95% confidence interval for the mean difference in the weight of these two varieties of tortoises?

A. $90 \pm 1.96\sqrt{\frac{(54.2)^2}{23}+\frac{(68.3)^2}{15}}$

B. $90 \pm 1.96\left(\frac{54.2}{\sqrt{23}}+\frac{68.3}{\sqrt{15}}\right)$

C. $90 \pm 2.145\sqrt{\frac{(54.2)^2}{23}+\frac{(68.3)^2}{15}}$

D. $90 \pm 2.145\left(\frac{54.2}{\sqrt{23}}+\frac{68.3}{\sqrt{15}}\right)$

E. $90 \pm 2.145\sqrt{\frac{(54.2)^2}{23}-\frac{(68.3)^2}{15}}$

4.

Researchers who are interested in whether or not vitamins contribute to an increase in energy found 35 volunteers to participate in a study. Each person was assigned to take a pill for 10 days and then take a test to evaluate their energy level. One month later they took a second pill for 10 days and took the same test to evaluate their energy level. The order in which they took the treatments (vitamins, placebo) was determined by a coin toss. The results for the tests are listed below.

	Placebo	Vitamins	Difference
Mean	27.3	28.75	1.45
Standard deviation	5.038	4.745	3.203

What is a 95% confidence level for the mean change in their energy level?

A. (–3.1713, 6.0713)
B. (–0.8843, 3.7843)
C. (–0.8428, 3.7428)
D. (0.3497, 2.5503)
E. (0.3889, 2.5111)

5.

A health care agency that works with the people in Western Africa believes that there is a significant difference in the life expectancy for men and women in this region. They randomly select the records for 100 males and then randomly select records for 100 females. Using the data from each person, a formula is used to compute the life expectancy for the individual. The results for the data are listed below:

	$\bar{x}$	s	n
Life Expectancy for Women	58.75	6.628	100
Life Expectancy for Men	55.4375	5.609	100

Which of the following statement(s) is true for a 95% confidence interval for the difference in mean life expectancies for men and women in Western Africa?

I. 95% of the time, the difference between the mean life expectancy between men and women in Western Africa will be between 1.60 and 5.03 years.
II. The confidence interval suggests that the mean life expectancy for women is greater than the life expectancy for men.
III. The true mean difference between the life expectancy for men and women is 3.313 years.

A. I only
B. II only
C. III only
D. I and II only
E. II and III only

6.

The board of directors at a very large manufacturing plant decides to determine a 99% confidence interval for the difference in the mean number of years on the job for management and plant workers. They randomly select 50 people from the management team and 50 people from the plant workers and then calculate the confidence interval for the mean difference in the number of years on the job. Which of the following statements is true about the confidence interval?

A. The sample distribution of the mean number of years on the job for the plant workers is likely to contain large outliers so the assumptions for the two-sample *t*-interval are not valid.
B. All of the assumptions are met and the confidence interval is valid.
C. The plant workers and management team are not independent so the assumptions for the two-sample *t*-interval are not valid.
D. The sampling distribution for number of years on the job for both management and plant workers are likely skewed so the assumptions for the confidence interval are not valid.
E. The confidence interval is too wide to capture the true mean difference in the mean number of years on the job for management and plant workers.

Free Response Questions:

7. One of the two ambulance services in a certain town responds to calls in the eastern half of the town, and the other ambulance responds to calls in the western half of the town. One of the hospital administrators believes that the two ambulances have different mean response times. Response time is measured by the difference between the time an emergency call comes into the dispatcher and the time the ambulance arrives at the scene of the emergency.

   Data were collected to investigate whether the hospital administrator's belief is correct. A random sample of 75 calls selected from the eastern ambulance service had a mean response time of 3.9 minutes with a standard deviation of 2.8 minutes. A random sample of 75 calls selected from the western ambulance service had a mean response time of 4.9 minutes with a standard deviation of 2.6 minutes.

   (a) Construct and interpret and 90% confidence interval for the difference in mean response times between the two ambulance services.

   (b) Does the confidence interval in part (a) support the hospital administrator's belief that the two ambulance services have different mean times? Explain.

8. A researcher believes that treating seeds with certain additives before planting can enhance the growth of plants. An experiment to investigate this is conducted in a greenhouse. From a large number of Jalapeno seeds, 36 seeds are randomly chosen and 2 are assigned to each of 18 containers. One of the 2 seeds is randomly selected and treated with the additive. The other seed serves as a control. Both seeds are then planted in the same container. The growth, in centimeters, of each of the 36 plants is measured after 30 days. These data were used to generate the partial computer output shown below. Graphical displays indicate that the assumption of normality is not unreasonable.

	N	Mean	StDev	SE Mean
Control	18	24.649	2.108	0.497
Treatment	18	26.017	3.375	0.795
Difference	18	-1.368	2.903	0.684

(a) Construct a confidence interval for the mean difference in growth, in centimeters, of the plants from the untreated and treated seeds. Be sure to interpret this interval.

(b) Based only on the confidence interval in part (a), is there sufficient evidence to conclude that there is a significant mean difference in growth of the plants from untreated seeds and the plants from treated seeds? Justify your conclusion.

9. The principal at McMillon High School, which enrolls only eleventh-grade students and twelfth-grade students, is interested in determining how much time students at that school spend on the internet each night. The table below shows the mean and standard deviation of the amount of time spent on the internet each night (in minutes) for a random sample of 18 eleventh-grade students and an independent random sample of 18 twelfth-grade students at this school.

	Mean	Standard Deviation
Eleventh-grade students	77.3	15.8
Twelfth-grade students	97.0	17.4

Based on dotplots of these data, it is not unreasonable to assume that the distribution of times for each grade was approximately normally distributed.

(a) Estimate the difference in mean times spent on the internet for all eleventh- and twelfth-grade students in this school using an interval. Be sure to interpret your interval.

(b) An assistant principal reasoned that a much narrower confidence interval could be obtained if the students were paired based on their responses; for example, pairing the eleventh-grade student and the twelfth-grade student with the highest number of minutes spent on the internet, the eleventh-grade student and twelfth-grade student with the next highest number of minutes spent on the internet, and so on. Is the assistant principal correct in thinking that matching students in this way and then computing a matched-pairs confidence interval for the mean difference in time spent on the internet is a better procedure than the one used in part (a)? Explain why or why not.