Normal Distributions

Skills, concepts, and reminders:

1. The Empirical Rule states that for an approximately normal distribution
   - 68% of the area is within 1 standard deviation of the mean.
   - 95% of the area is within 2 standard deviations of the mean.
   - 99.7% of the area is within 3 standard deviations of the mean.
2. Calculating *z*-scores:
   - A *z*-score represents the number of standard deviations a data point is above or below the mean.
   - The formula to determine a *z*-score is $z = \frac{x-\mu}{\sigma}$, where x represents a variable with a normal distribution $N(\mu, \sigma)$, with mean μ and standard deviation σ.
   - Example:
     The distribution of heights of adult males is approximately normal with a mean of 68 inches and a standard deviation of 2.4 inches. What percent of men are at least 75 inches?
     $P(X \geq 75) = P\left(z \geq \frac{75-68}{2.4}\right)$
     $= 1 - P(z < 2.917)$
     $= 1 - 0.9982 = 0.0018$ or 0.18% of men are at least 75 inches tall.
   - Example:
     What is the lower quartile of the distribution for the heights of men?
     $z = -0.67$ based on an tail area of 25%; $-0.67 = \frac{x-68}{2.4}$; $x = 69.61$ inches; The lower quartile of the distribution for the heights of men is about 69.61 inches.
3. Central Limit Theorem
   - The sampling distribution of sample means becomes approximately normal as the sample size *n* increases, regardless of the shape of the population.
   - The mean of the sampling distribution of sample means is equal to the mean of the population.
   - The standard deviation of the sampling distribution of sample means gets smaller as sample size increases according to the equation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
   - Example:
     Suppose that the cans of soda that are filled by a particular bottling company are filled to an average volume of 12 ounces with a standard deviation 0.2 ounces. The distribution volume is approximately normally distributed. What is the probability that a random sample of 4 cans has an average volume of at least 12.2 ounces?
     $z = \frac{12.2-12}{\frac{0.2}{\sqrt{4}}} = 2$; $P(x > 12.2) = P(z > 2) = 0.023$;
     Less than 2.3% of samples of 4 cans will have an average volume of at least 12.2 oz.

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Multiple Choice Questions:

1.

Lauren is enrolled in a very large college calculus class. On the first exam, the class mean was 75 and the standard deviation was 10. On the second exam, the class mean was 70 and the standard deviation was 15. Lauren scored 85 on both exams. Assuming the scores on each exam were approximately normally distributed, on which exam did Lauren score better relative to the rest of the class?

A. She scored much better on the first exam.
B. She scored much better on the second exam.
C. She scored about equally well on both exams.
D. It is impossible to tell because the class size is not given.
E. It is impossible to tell because the correlation between the two sets of exam scores is not given.

2.

The lengths of individual shellfish in a population of 10,000 shellfish are approximately normally distributed with mean 10 centimeters and standard deviation 0.2 centimeter. Which of the following is the shortest interval that contains approximately 4,000 shellfish lengths?

A. 0 cm to 9.949 cm
B. 9.744 cm to 10 cm
C. 9.744 cm to 10.256 cm
D. 9.895 cm to 10.105 cm
E. 9.9280 cm to 10.080 cm

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3.

The heights of a population of adult male red kangaroos are approximately normally distributed with a mean height of 1.20 meters and a standard deviation of 0.10 meters. The heights of a population of adult male grey kangaroos are approximately normally distributed with a mean height of 1.75 meters and a standard deviation of 0.25 meters. A certain adult male grey kangaroo is 2.05 meters tall. This kangaroo would have the same standardized height (z-score) as an adult male red kangaroo whose height, in meters, is equal to which of the following?

A. 1.08
B. 1.20
C. 1.30
D. 1.32
E. 1.45

4.

The distribution of heights of a simple random sample of 400 young children is approximately normal and produced the following summary statistics.

Mean	47 inches
Median	47 inches
Standard deviation	2 inches
First Quartile	45.7 inches
Third Quartile	48.3 inches

Which of the following statements is false?

A. About 64 children in the sample have heights that are greater than 49 inches.
B. About 200 children in the sample have heights that are between 45 and 49 inches.
C. About 200 children in the sample have heights that are greater than 47 inches.
D. About 336 children in the sample have heights that are greater than 45 inches.
E. About 380 children in the sample have heights that are between 43 and 51 inches

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5.

The distribution of the diameters of a particular variety of cantaloupe is approximately normal with a standard deviation of 0.7 inches. If the diameter of a cantaloupe is at the 73rd percentile, how does this diameter compare to the mean?

A. 0.511 inches below the mean
B. 0.427 inches below the mean
C. 0.511 inches above the mean
D. 0.427 inches above the mean
E. 0.610 inches above the mean

6.

The weights of adult male Chinook salmon caught in the rivers along the western coast of Washington are normally distributed with a mean of 25 kilograms and standard deviation 8 kilograms. Suppose regulations require that only 3 salmon can be caught per day by any one fisherman. Assuming that on his next trip Jim catches three salmon, what is the approximate probability that the average weight of the fish on his stringer is at least 30 kilograms?

A. 0.03
B. 0.14
C. 0.27
D. 0.41
E. 0.45

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7.

The heights of mature maple trees are approximately normally distributed with a mean of 80 feet and standard deviation 12.5 feet. What proportion of mature maple trees are between 65 and 90 feet?

A. 0.1151
B. 0.2119
C. 0.6731
D. 0.7881
E. 0.8849

8.

Golf balls must meet a certain standard of distance traveled in order to be used in a professional tournament. When the ball is hit by a mechanical device, under specific calibration, the ball may not travel farther than 291.2 yards in the air. From past data, a certain manufacturer has determined that the distances traveled for the balls it produces are normally distributed with a mean of 290 yards. What standard deviation, in yards, should the manufacturer require if they want 99% of balls they manufacture to meet the tournament standard?

A. 0.47
B. 0.52
C. 0.94
D. 1.04
E. 1.20

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Free Response Questions:

9. Tennis balls must meet certain specifications in order to be used in official tournament play. One of these specifications is rebound. For tournaments that are being played in "high altitude" conditions, a ball dropped from a height of 100 inches must rebound to a height between 48 and 53 inches. A particular manufacturer has determined that the rebound heights for balls it manufactures for high altitude conditions are normally distributed with standard deviation of 1.1 inches. This manufacturer has a new process that allows it to set the mean rebound of these balls.

   (a) If the manufacturer initially sets the mean rebound height to be 50 inches, what proportion of balls produced by this method will not meet the tournament specifications for rebound?

   (b) Assume that the mean distance is set to 50 inches and that nine balls are independently tested. What is the probability that no more than one of the balls fail to meet the rebound specifications?

   (c) What mean rebound height should the manufacturer use to maximize the proportion of balls it sells that meet the specifications for high altitude rebound? Justify your answer.

   (d) What is the maximum proportion of balls produced by this manufacturing process that will meet the high altitude rebound specifications?

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10. A large wildlife rehabilitation center recently released over 500 birds of an almost extinct species into a large national park. The distribution of the wingspans of these birds is approximately normal.

(a) The rehabilitation center claims that the mean wingspan of these birds is 28 centimeters. If the claim is true which of the following would be more likely?

- A random sample of 10 birds with a mean wingspan that is less than 25 cm

or

- A random sample of 40 birds with a mean wingspan that is less than 25 cm

Justify your answer.

(b) Suppose the standard deviation of the sampling distribution of the sample means for a random sample of size 40 is 1.4 cm. If the mean wingspan of the birds is 28 cm, use the normal distribution to compute the probability that a random sample of 40 birds will have a mean wingspan greater than 31 cm.

(c) Suppose the distribution of wingspans for this species of bird in the park was nonnormal but had the same mean and standard deviation. Would it still be appropriate to use the normal distribution to compute the probability in part (b) Justify your answer.

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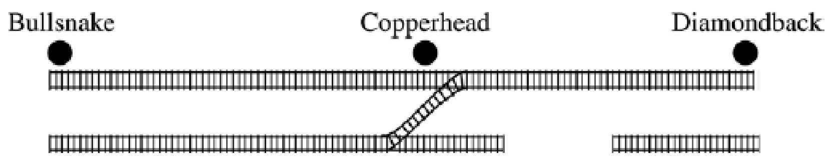
11. Flooding has washed out one of the tracks of the Snake Gulch Railroad. The railroad has two parallel tracks from Bullsnake to Copperhead, but only one useable track from Copperhead to Diamondback, as shown in the figure below. Having only one useable track disrupts the usual schedule. Until it is repaired, the washed-out track will remain unusable. If the train leaving Bullsnake arrives at Copperhead first, it has to wait until the train leaving Diamondback arrives at Copperhead.



Every day at noon a train leaves Bullsnake heading for Diamondback and another leaves Diamondback heading for Bullsnake.

Assume that the length of time, X, it takes the train leaving Bullsnake to get to Copperhead is normally distributed with a mean of 170 minutes and a standard deviation of 20 minutes.

Assume that the length of time, Y, it takes the train leaving Diamondback to get to Copperhead is normally distributed with a mean of 200 minutes and a standard deviation of 10 minutes.

These two travel times are independent.

(a) What is the distribution of $Y - X$?

(b) Over the long run, what proportion of the days will the train from Bullsnake have to wait at Copperhead for the train from Diamondback to arrive?

(c) How long should the Snake Gulch Railroad delay the departure of the train from Bullsnake so that the probability that it has to wait is only 0.01?

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