



1-Proportion Confidence Intervals

Requirements for complete responses to free response questions that require a 1-Proportion Confidence interval:

1. Identify the population parameter of interest, the type of inferential procedure to be used, and the confidence level.
 - A confidence interval is used to estimate the value of an unknown *parameter*.
 - The *confidence level* gives the percent of the intervals produced by the procedure you used that capture the true value of the parameter.
 - The population parameter of interest must be defined in context of the problem and must be identified using p for population proportion.
2. State and verify whether the conditions (assumptions) needed for this procedure are met.
 - The conditions for 1-Proportion z-Intervals are as follows:
 - The sample must be a random sample from the population of interest. Usually this is stated in the prompt. You must state this condition and verify whether or not this condition is met.
 - The sample must be large. To verify this condition, check that $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$. (Note: \hat{p} is used instead of p because we do not know the value of p . We are trying to estimate it!) You must show your work to receive credit for verifying this condition.
 - The individual observations are independent. When sampling without replacement, the population must be at least 10 times greater than the sample size.
3. Write the formula for the confidence interval and determine the interval.
 - Every confidence interval computation follows the same general form (given on the formula sheet):
(Sample statistic) \pm (distribution critical value) \times (standard deviation of statistic)
4. Interpret the interval in the context of the problem.
 - Include a statement that links the *confidence level* to the interval.
 - When you interpret a confidence interval, be very careful that you do not associate the probability with the parameter. Keep in mind that the value of the parameter does not vary, so there is no chance associated with its value. The sample is what varies!
 - It is risky to try to interpret a confidence interval in a creative way. Learn a good confidence interval interpretation and stick to it! A suggested writing template for the interval is:
We are _____% confident that the true proportion of “context of problem” is between _____ and _____ (units).

If you are asked to interpret the confidence level, the following is an appropriate interpretation of confidence level:

_____ % of the intervals produced using this method will capture the true proportion of “context of problem.”

Selecting a sample size:

When planning a study, the sample size chosen allows us to estimate a population proportion within a given margin of error.

- The formula to determine the sample size based on the desired confidence level is

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq \text{ME}$$

- When you calculate the sample size needed to obtain a certain margin of error, the answer must be an integer equal to or greater than the value you obtain algebraically. (round to the greatest whole number)
- Increasing the confidence level results in a larger margin of error (if all else remains the same).

Increasing the sample size results in a smaller margin of error (if all else remains the same).

Multiple Choice Questions:

1.

USA Today reported that speed skater Bonnie Blake had “won the USA’s heart,” according to a *USA Today*/CNN/Gallup poll conducted on the final Thursday of the 1994 Winter Olympics. When asked who was the hero of the Olympics, 65 percent of the respondents chose Blair, who won five gold medals.

The poll of 615 adults, done by telephone, had a margin of error of 4 percent. Which of the following statements best describes what is meant by the 4 percent margin of error?

- A. About 4 percent of adults were expected to change their minds between the time of the poll and its publication in *USA Today*.
- B. About 4 percent of adults did not have telephones.
- C. About 4 percent of the 615 adults polled refused to answer.
- D. Not all of the 615 adults knew anything about the Olympics.
- E. The difference between the sample percentage and the population percentage is likely to be less than 4 percent.

2.

A 95 percent confidence interval of the form $\hat{p} \pm E$ will be used to obtain an estimate for an unknown parameter p . If \hat{p} is the sample proportion and E is the margin of error, which of the following is the smallest sample size that will guarantee a margin of error of at most 0.08?

- A. 25
- B. 100
- C. 175
- D. 250
- E. 625

3.

A survey was conducted to determine what percentage of college seniors would have chosen to attend a different college if they had known then what they know now. In a random sample of 100 seniors, 34 percent indicated that they would have attended a different college. A 90 percent confidence interval for the percentage of all seniors who would have attended a different college is

- A. 24.7% to 43.3%
- B. 25.8% to 42.2%
- C. 26.2% to 41.8%
- D. 30.6% to 37.4%
- E. 31.2% to 36.8%

4.

Courtney has constructed a cricket out of paper and rubber bands. According to the instructions for making the cricket, when it jumps it will land on its feet half of the time and on its back the other half of the time. In the first 50 jumps, Courtney's cricket landed on its feet 35 times. In the next 10 jumps, it landed on its feet only twice. Based on this experience, Courtney can conclude that

- A. the cricket was due to land on its feet less than half the time during the final 10 jumps, since it had landed too often on its feet during the first 50 jumps
- B. a confidence interval for estimating the cricket's true probability of landing on its feet is wider after the final 10 jumps than it was before the final 10 jumps
- C. a confidence interval for estimating the cricket's true probability of landing on its feet after the final 10 jumps is exactly the same as it was before the final 10 jumps
- D. a confidence interval for estimating the cricket's true probability of landing on its feet is more narrow after the final 10 jumps than it was before the final 10 jumps
- E. a confidence interval for estimating the cricket's true probability of landing on its feet based on the initial 50 jumps does not include 0.2, so there must be a defect in the cricket's construction to account for the poor showing in the final 10 jumps

5.

A researcher has conducted a survey using a simple random sample of 50 registered voters to create a confidence interval to estimate the proportion of registered voters favoring the election of a certain candidate for mayor. Assume that the sample proportion does not change. Which of the following best describes the anticipated effect on the width of the confidence interval if the researcher were to survey a random sample of 200, rather than 50, registered voters?

- A. The width of the new interval would be about one-fourth the width of the original interval.
- B. The width of the new interval would be about one-half the width of the original interval.
- C. The width of the new interval would be about the same width as the original interval.
- D. The width of the new interval would be about twice the width of the original interval.
- E. The width of the new interval would be about four times the width of the original interval.

6.

A city is interested in building a waste management facility in a certain area. One hundred randomly selected residents from this area were asked, "Do you support the city's decision to build a waste management facility in your area?" Of the 100 residents interviewed, 54 said

no, 4 said yes, and 42 had no opinion. A large sample z -confidence interval, $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$,

was constructed from these data to estimate the proportion of this area's residents who support building a waste management facility in their area. Which of the following statements is correct for this confidence interval?

- A. This confidence interval is valid because a sample size of more than 30 was used.
- B. This confidence interval is valid because each area resident was asked the same question.
- C. This confidence interval is valid because no conditions are required for constructing a large sample confidence interval for a proportion.
- D. This confidence interval is not valid because the quantity $n\hat{p}$ is too small.
- E. This confidence interval is not valid because "no opinion" was included as a response category for the question.

Free Response Questions:

7. A husband and wife, Mike and Lori, share a digital music player that has a feature that randomly selects which song to play. A total of 2,384 songs were loaded onto the player, some by Mike and the rest by Lori. Suppose that when the player was in the random-selection mode, 13 of the first 50 songs selected were songs loaded by Lori.

(a) Construct and interpret a 90 percent confidence interval for the proportion of songs on the player that were loaded by Lori.

(b) Mike and Lori are unsure about whether the player samples the songs with replacement or without replacement when the player is in random-selection mode. Explain why this distinction is not important for the construction of the interval in part (a).

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Student Study Session

8. Each person in a random sample of 1,026 adults in the United States was asked the following question. “Based on what you know about the Social Security system today, what would you like Congress and the President to do during this next year?”

The response choices and the percentages selecting them are shown below.

Completely overhaul the system	19%
Make some major changes	39%
Make some minor adjustments	30%
Leave the system the way it is now	11%
No opinion	1%

- (a) Find a 95% confidence interval for the proportion of all United States adults who would respond “Make some major changes” to the question. Give an interpretation of the confidence interval and give an interpretation of the confidence level.

- (b) An advocate for leaving the system as it is now commented, “Based on this poll, only 39% of adults in the sample responded that they want some major changes made to the system, while 41% responded that they want only minor changes or no changes at all. Therefore, we should not change the system.” Explain why this statement, while technically correct, is misleading.

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Student Study Session

9. During a flu vaccine shortage in the United States, it was believed that 45 percent of vaccine-eligible people received flu vaccine. The results of a survey given to a random sample of 2,350 vaccine-eligible people indicated that 978 of the 2,350 people received flu vaccine.
- (a) Construct a 99 percent confidence interval for the proportion of vaccine-eligible people who had received flu vaccine. Use your confidence interval to comment on the belief that 45 percent of the vaccine-eligible people had received flu vaccine.
- (b) Suppose a similar survey will be given to vaccine-eligible people in Canada by Canadian health officials. A 99 percent confidence interval for the proportion of people who will have received flu vaccine is to be constructed. What is the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02.