1-Proportion Hypothesis Tests

Requirements for complete responses to free response questions that require 1-Proportion Hypothesis tests:

1. Identify the population parameter of interest, the type of inferential procedure to be used, the null and the alternative hypothesis, and the significance level of the test.
   - The population parameter of interest must be defined in context of the problem and must be identified using *p* for population proportion.
   - The null hypothesis is always an "equals" statement.
   - The alternative hypothesis is always an inequality statement (use $<$, or $>$, or $\neq$)
   - Remember that the significance level of the test α is the probability of Type I error.
2. State and verify whether the conditions (assumptions) needed for this procedure are met.
   - The conditions for 1-Proportion *z*-tests are as follows:
     - The sample must be a random sample from the population of interest. Usually this is stated in the prompt. You must state this condition and verify whether it's met.
     - The sample must be large. To verify this condition check that $np \geq 10$ and $n(1-p) \geq 10$. You must show your work to receive credit for verifying this condition.
     - The individual observations are independent. When sampling without replacement, the population must be at least 10 times greater than the sample size.
3. Show the results of the test including the formula for the test statistic, the value of the test statistic, *p*-value, and a picture of the sampling distribution showing the hypothesized mean, the test statistic, and the associated *p*-value shaded.
   - Every test statistic computation follows the same general form (on the formula sheet):

$$\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$$

4. State the conclusion in context of the situation.
   - Include a statement linking the *p*-value to the significance level and the decision about the null hypothesis (fail to reject or reject) and a second statement about whether or not there is evidence to support the alternative hypothesis. The alternative should always be stated in context. Remember you never accept the null hypothesis!

Other things to remember about Hypothesis testing in general:

- The *p*-value of a test gives the probability that the observed outcome, or one even more different from the claim of the null hypothesis, would occur just by chance if the null hypothesis were true.
- Small *p*-values indicate that there is little chance that the observed outcome, or one even more different from the claim of the null hypothesis, would occur just by chance if the null hypothesis were true. Small *p*-values lead us to reject the null hypothesis.
- Large *p*-values indicate that the observed outcome, or one even more different from the claim of the null hypothesis, is not rare and would be likely to occur just by chance if the null hypothesis were true. So large *p*-values lead us to conclude that the null hypothesis should not be rejected.

Reality

Decision		H_0 is true	H_0 is false
	Fail to reject H_0	Correct decision	Type II error
	Reject H_0	Type I error	Power (correct decision)

- A *Type I error* occurs when you reject a null hypothesis that is true.
- A *Type II error* occurs when you fail to reject a null hypothesis that is not true.
- The *power* of a test is the probability that a test will reject a false null hypothesis. It makes sense to think of the *power* of the test as how *sensitive* the test is to detect a false null hypothesis.
  - $\text{Power} = 1 - P(\text{Type II error})$
  - Power increases when sample size increases (the standard error decreases); power increases when the α-level of a test increases (the test is more likely to reject the null hypothesis at the 10% level than at the 5% level);

Writing templates for Hypothesis Testing:

Conclusion for a hypothesis test:
Since the *p*-value is (less/greater) than alpha, (show inequality statement), I (reject/ fail to reject) the null hypothesis. There (is/ is not) statistically significant evidence that (state the alternative hypothesis in context.)

Interpretation of *p*-value:
The probability of observing a sample (statistic of interest) as extreme or more extreme than (state the observed value), assuming (state the null hypothesis in context) is true, is (state the *p*-value).

Type I error:
A type I error would be rejecting the null hypothesis that (state the null in context) when in fact it is true.

Type II error:
A type II error would be failing to reject the null hypothesis that (state the null in context) when in fact (state the alternative in context).

Multiple Choice Questions:

1.

A manufacturer of rubber bands claims that *p*, the proportion of its rubber bands that break when stretched to a length of up to 8 inches, is no more than 0.03. Some customers have complained that the rubber bands are breaking more frequently. If the customers want to conduct an experiment to test the manufacturer's claim, which of the following hypotheses would be appropriate?

A. $H_O: p \neq 0.03, \quad H_A: p = 0.03$
B. $H_O: p = 0.03, \quad H_A: p > 0.03$
C. $H_O: p = 0.03, \quad H_A: p \neq 0.03$
D. $H_O: p = 0.03, \quad H_A: p < 0.03$
E. $H_O: p < 0.03, \quad H_A: p = 0.03$

2.

A consulting statistician reported the results from a learning experiment to a psychologist. The report stated that on one particular phase of the experiment a statistical test result yielded a *p*-value of 0.24. Based on this *p*-value, which of the following conclusions should the psychologist make?

A. The test was statistically significant because a *p*-value of 0.24 is greater than a significance level of 0.05.
B. The test was statistically significant because $p = 1 - 0.24 = 0.76$ and this is greater than a significance level of 0.05.
C. The test was not statistically significant because 2 times $0.24 = 0.48$ and that is less than 0.5.
D. The test was not statistically significant because, if the null hypothesis is true, one could expect to get a test statistic at least as extreme as that observed 24% of the time.
E. The test was not statistically significant because, if the null hypothesis is true, one could expect to get a test statistic at least as extreme as that observed 76% of the time.

3.

A city is interested in building a water park in a certain area. Thirty-five randomly selected residents from this area were asked, “Do you support the city’s proposal to build a water park in your area?” Of the 35 residents interviewed, 31 said yes, 3 said no, and 1 had no opinion. A large sample *z*-test, $H_0 : p = 0.80$; $H_a : p > 0.80$ was used to determine if there is evidence that more than 80 percent of the area supported building the water park. Which of the following statements is correct for this hypothesis test?

A. This hypothesis test is valid because a sample size of more than 30 was used.
B. This hypothesis test is valid because each area resident was asked the same question.
C. This hypothesis test is valid because no conditions are required for conducting a large sample hypothesis test for a proportion.
D. This hypothesis test is not valid because the quantity $n(1-p)$ is too small.
E. This hypothesis test is not valid because “no opinion” was included as a response category for the question.

4.

A cancer research scientist wants to test the null hypothesis that a new chemotherapy is not effective. Under which of the following conditions would a Type II error occur?

A. The scientist concludes that the chemotherapy is effective when it actually is.
B. The scientist concludes that the chemotherapy is effective when it actually is not.
C. The scientist concludes that the chemotherapy is not effective when it actually is.
D. The scientist concludes that the chemotherapy is not effective when it actually is not.
E. The scientist concludes that the chemotherapy is not effective when there is statistical evidence that it is.

5.

In a large national park, several varieties of venomous and non-venomous snakes can be found. Data were obtained from a sample of 100 snakes on species and size. The National Park Service rangers want to see if the data provide convincing evidence that the proportion of venomous snakes in the park is above a permissible level. Of the following which information would NOT be expected to be a part of the process of correctly conducting a hypothesis test, at the 0.05 level of significance?

A. Verifying that the sample is large enough park using $np \geq 10$ and $n(1-p) \geq 10$.
B. Assuming that the sample of snakes is representative of the overall population of snakes in the park.
C. Using a *z*-statistic to carry out the test
D. Given that the *p*-value is less than 0.05, failing to reject the null hypothesis and concluding there is not evidence that the proportion of venomous snakes in the park is above the permissible level.
E. Accepting a 5 percent risk of committing a Type I error.

6.

In Texas, state law requires that for the elementary grades K-4, the student-to-teacher ratio be less than 22:1 for any regular classroom setting. Schools may request waiver to exceed this ratio. A hypothesis test is to be done to determine if the proportion of schools needing to request a waiver is above 25% using a random sample of 85 schools. What conclusion should be drawn if 32 of the schools surveyed are going to request a waiver and a significance level of 5% is used?

A. Since the *p*-value is less than alpha, the null hypothesis should not be rejected. There is not significant evidence that the true proportion of schools needing a waiver is greater than 25%.
B. Since the *p*-value is greater than alpha, the null hypothesis should be rejected. There is significant evidence that the true proportion of schools needing a waiver is greater than 25%.
C. Since the *p*-value is less than alpha, the null hypothesis should be rejected. There is significant evidence that the true proportion of schools needing a waiver is greater than 25%.
D. Since $\hat{p} = 37.6\%$ is greater than 25%, the null hypothesis should be rejected. There is significant evidence that the true proportion of schools needing a waiver is greater than 25%.
E. Since $\hat{p} = 37.6\%$ is greater than 5%, the null hypothesis should not be rejected. There is not significant evidence that the true proportion of schools needing a waiver is greater than 25%.

Free Response Questions:

7. Some bottles of a certain brand of flavored iced tea include a code for a free music download on the lid of the bottle. The company that makes the flavored tea claims that a code can be found in 25% percent of the bottles. However, based on their experiences drinking this tea at home, a group of students believes that the proportion of bottles with codes is less than 0.25. This group of students purchased 82 bottles of the tea to investigate the company's claim. The students found a total of 17 codes for free music downloads in the 82 bottles.

   Suppose it is reasonable to assume that the 82 bottles purchased by the students are a random sample of all bottles of this tea. Based on this sample, is there support for the students' belief that the proportion of bottles with codes is less than 0.25? Provide statistical evidence to support your answer.

8. A new window is being advertised to withstand hurricane force winds in excess of 155 miles per hour at a rate of 85% success. In an experiment on these windows, a random sample of 200 of these windows were placed in a wind tunnel and exposed to winds exceeding 155 miles per hour. After the test, 34 of the windows had broken due to the pressure from the wind speed.

   A test of hypothesis was conducted on the following hypothesis.

   H_0 : The success rate of the new style of window is 85%.

   H_a : The success rate of the new style of window is less than 85%.

   The test resulted in a *p*-value of 0.2141.

   (a) Interpret the *p*-value in context of this study.

   (b) Based on this *p*-value, what conclusion should be drawn in the context of the study? Use a significance level of $\alpha = 0.05$.

   (c) Based on your conclusion in part (b) which type of error, Type I or Type II, could have been made? What is one potential consequence of this error?

9. A large university provides housing for 10 percent of its graduate students to live on campus. The university's housing office thinks that the percentage of graduate students looking for housing on campus may be more than 10 percent. The housing office decides to survey a random sample of graduate students, and 62 or the 481 respondents say that they are looking for housing on campus.

   (a) On the basis of the survey data, would you recommend that the housing office consider increasing the amount of housing on campus available to graduate students? Give appropriate evidence to support your recommendations.

(b) In addition to the 481 graduate students who responded to the survey, there were 19 who did not respond. If these 19 had responded, is it possible that your recommendation would have changed? Explain.